[6]Stucheli, A., and F. Widmer, "Nicht-Isotherme riesel-filmstromung einer hochviskosen flussigkeit mit stork temperaturabhangiger viskositat," warme-und stoffubertragung, Vol. 11, No. 1, 1978, pp. 91–101.

Approximate Solution of the Thermal-Entry-Length Fluid Flow and Heat Transfer Characteristics in Annuli with Blowing at the Walls

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Nomenclature

C_P = specific heat
D_h = hydraulic diameter, $= 2\,(R_o - R_i)$
G = $(1 - K^2)/\ln(1/K)$
H = $(1 - K^4)/(1 - K^2)$
h = convective heat transfer coefficient
K = ratio of the inner radius to the outer radius of the annulus, $= R_i/R_o$
k = thermal conductivity of the fluid
Nu = Nusselt number, $= h\,D_h/k$
P^+ = dimensionless pressure, $= (P - P_e)/\rho\, w_{em}^2$
P = pressure
Pr = Prandtl number, $= \mu\, C_p/k$
Re_i = injection radial Reynolds number at the inner wall, $= R_i\, v_i/\nu$
Re_o = injection radial Reynolds number at the outer wall, $= R_o\, v_o/\nu$
R_i = inner radius of the annulus
R_o = outer radius of the annulus
r = radial direction
r^+ = dimensionless radial distance, $= r/R_o$
T = temperature
v = radial component of velocity
v^+ = dimensionless radial velocity, $= R_o\, v/\nu$
W^+ = dimensionless axial velocity based on the local mean velocity, $= w^+/w_m^+$
w = axial component of velocity
w_{em} = mean value of fully developed velocity profile at entrance to heated length
w^+ = dimensionless axial velocity, $= w/w_{em}$
z = axial direction
z^+ = dimensionless axial distance, $= \nu z/R_o^2\, w_{em}$
θ_l = dimensionless temperature, $= (T - T_e)/(T_l - T_e)$
θ_{ml} = dimensionless mean temperature
μ = absolute viscosity
ν = kinematic viscosity
ρ = density

Subscripts
e = entrance to the annulus
i = inner wall
l = dummy subscript: o, i
m = mixed mean
o = outer wall

Superscript
$+$ = dimensionless variable

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Introduction

THIS Note is a continuation of an effort to study the convective heat transfer in annular porous passages.[1] The subject of fluid flow in annuli with blowing and suction at the walls has received attention recently because of applications of the concentric annular heat pipe.[2,3] The numerical solutions by Faghri[1] were obtained using finite difference methods for thermally and hydrodynamically developing laminar flow in annular passages with blowing and suction at the walls. In the present study, the thermal-entry-length heat transfer characteristics of annuli with blowing at the walls have been approximated by using the fully developed axial and radial velocity profiles that were obtained analytically and used in a simple numerical scheme to solve the energy equation. An equation for the pressure drop in annuli with blowing or impermeable walls is also derived by solving the axial momentum equation with the fully developed axial and radial velocity profiles.

Analysis

The nondimensional equations associated with the flow in an annulus with uniformly porous walls are the continuity, momentum, and energy equations, as presented for the case of a constant-property, laminar, incompressible, steady flow with no transverse pressure gradient or longitudinal diffusion:

$$r^+ \frac{\partial w^+}{\partial z^+} + \frac{\partial (r^+ v^+)}{\partial r^+} = 0 \tag{1}$$

$$w^+ \frac{\partial w^+}{\partial z^+} + v^+ \frac{\partial w^+}{\partial r^+} = -\frac{\partial P^+}{\partial z^+} + \frac{\partial^2 w^+}{\partial r^{+2}} + \frac{1}{r^+}\frac{\partial w^+}{\partial r^+} \tag{2}$$

$$w^+ \frac{\partial \theta}{\partial z^+} + v^+ \frac{\partial \theta}{\partial r^+} = \frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial r^{+2}} + \frac{1}{r^+}\frac{\partial \theta}{\partial r^+}\right] \tag{3}$$

The hydrodynamic boundary conditions with a fully developed velocity profile at the entrance to the heated length are

$$w^+(0, r^+) = \frac{2[1 - r^{+2} + G \ln(r^+)]}{(H - G)}$$

$$w^+(z^+, K) = w^+(z^+, 1) = 0$$

$$v^+(z^+, K) = \frac{Re_i}{K}$$

$$v^+(z^+, 1) = Re_o$$

$$P^+(0, r^+) = 0$$

where $Re_i > 0$ and $Re_o < 0$ are the injection radial Reynolds numbers at the inner and outer walls, respectively. $Re_i = Re_o = 0$ corresponds to an impermeable wall. The fully developed entrance velocity profile was analytically derived for annuli with impermeable walls.[4]

Consider the general thermal boundary condition set that corresponds to a step temperature increase or decrease at the inner and outer walls:

$$T = T_e \qquad \text{everywhere for } z^+ \le 0 \tag{4a}$$

$$T = T_i \qquad r^+ = K, \qquad z^+ > 0 \tag{4b}$$

$$T = T_o \qquad r^+ = 1, \qquad z^+ > 0 \tag{4c}$$

Let θ_o be the solution to Eq. (3) in which the outer wall experiences a step temperature and the inner wall remains at the inlet temperature:

$$\theta_o = 0 \qquad \text{everywhere for } z^+ \le 0 \text{ and}$$

$$r^+ = K, \qquad z^+ > 0 \tag{5a}$$

$$\theta_o = 1 \qquad r^+ = 1, \qquad z^+ > 0 \tag{5b}$$

Similarly, let θ_i be the solution to Eq. (3) in which the inner wall undergoes a step temperature and the outer wall remains at the inlet temperature:

$$\theta_i = 0 \qquad \text{everywhere for } z^+ \le 0 \text{ and}$$

$$r^+ = 1, \qquad z^+ > 0 \tag{6a}$$

$$\theta_i = 1 \qquad r^+ = K, \qquad z^+ > 0 \tag{6b}$$

The solution of Eq. (3) satisfying the general thermal boundary conditions (4) using Eqs. (5) and (6) is

$$T(r^+, z^+) = (T_o - T_e)\,\theta_o(r^+, z^+) + (T_i - T_e)\,\theta_i(r^+, z^+) + T_e \tag{7}$$

With prescribed inner and outer wall temperatures, the Nusselt numbers for the inner and outer walls based on the hydraulic diameter are defined in terms of the dimensionless wall heat fluxes by the following relations:

$$Nu_{oo} = \frac{2(1-K)\,(\mathrm{d}\theta_o/\mathrm{d}r^+)|_{r^+=1}}{1-\theta_{mo}} \tag{8}$$

$$Nu_{io} = \frac{2(1-K)\,(\mathrm{d}\theta_o/\mathrm{d}r^+)|_{r^+=K}}{\theta_{mo}} \tag{9}$$

$$Nu_{oi} = \frac{-2(1-K)\,(\mathrm{d}\theta_i/\mathrm{d}r^+)|_{r^+=1}}{\theta_{mi}} \tag{10}$$

$$Nu_{ii} = \frac{-2(1-K)\,(\mathrm{d}\theta_i/\mathrm{d}r^+)|_{r^+=K}}{1-\theta_{mi}} \tag{11}$$

where

$$\theta_{ml} = \frac{2}{(1-K^2)\,w_m^+}\int_K^1 w^+\theta_l r^+\,\mathrm{d}r^+$$

represents, in effect, the dimensionless mixed-mean temperature. In Eqs. (8–11), the first subscript on the Nusselt numbers refers to the wall at which the heat flux is evaluated and the second subscript refers to the wall that is being heated. For example, Nu_{io} refers to the Nusselt number at the inner wall when the outer wall is being heated. The Nusselt numbers of the general case corresponding to boundary conditions (4) are

$$Nu_o = \frac{(T_o - T_e)(1-\theta_{mo})\,Nu_{oo} - (T_i - T_e)\,\theta_{mi}\,Nu_{oi}}{(T_o - T_e)(1-\theta_{mo}) - (T_i - T_e)\,\theta_{mi}} \tag{12}$$

$$Nu_i = \frac{(T_o - T_e)\,\theta_{mo}\,Nu_{io} - (T_i - T_e)(1-\theta_{mi})\,Nu_{ii}}{(T_o - T_e)\,\theta_{mo} - (T_i - T_e)(1-\theta_{mi})} \tag{13}$$

The dimensionless conservation of mass equation (1) can be solved for the dimensionless radial velocity by integrating with respect to r^+:

$$v^+ = \frac{Re_i}{r^+} - \frac{1}{r^+}\int_K^{r^+} r^+\frac{\partial w^+}{\partial z^+}\,\mathrm{d}r^+ \tag{14}$$

Faghri[1] found that the vapor flow in annuli with blowing becomes fully developed in a very short distance from the entrance, after which the dimensionless axial velocity profile based on the local mean axial velocity, $W^+ = w^+/w_m^+$, becomes similar to the case of impermeable walls. Therefore, the dimensionless axial velocity profile for impermeable walls can be found analytically and used with the appropriate mean axial velocity to determine the axial velocity profile in an annulus with porous walls. The dimensionless axial velocity for fully developed flow in an annulus with impermeable walls is

$$W^+ = \frac{w^+}{w_m^+} = \frac{2[1 - r^{+2} + G\,\ln(r^+)]}{(H-G)} \tag{15}$$

The local mean axial velocity at any point along the length of the pipe can be found by a global mass balance:

$$w_m^+ = 1 + \frac{2z^+(Re_i - Re_o)}{(1-K^2)} \tag{16}$$

Substituting Eqs. (15) and (16) into Eq. (14) and integrating results in an expression for the radial velocity in the fully developed region

$$v^+ = \frac{Re_i}{r^+} - \frac{4(Re_i - Re_o)}{r^+(1-K^2)(H-G)}\left\{\frac{r^{+2}}{2} - \frac{r^{+4}}{4} + \frac{Gr^{+2}}{4}[2\ln(r^+) - 1] - \frac{K^2}{2} + \frac{K^4}{4} - \frac{GK^2}{4}[2\ln(K) - 1]\right\} \tag{17}$$

Equations (15–17) were used to analytically solve the conservation of momentum Equation (2) for the pressure drop along the annulus with porous walls in the hydrodynamically fully developed region. The conservation of momentum equation is integrated as follows:

$$\frac{\partial P^+}{\partial z^+}\left(\frac{1}{2} - \frac{K^2}{2}\right) = \left(r^+\frac{\partial w^+}{\partial r^+}\right)\Big|_K^1 - \int_K^1 r^+\left(w^+\frac{\partial w^+}{\partial z^+} + v^+\frac{\partial w^+}{\partial r^+}\right)\mathrm{d}r^+$$

The dimensionless pressure in the fully developed region is given by

$$P^+ = 4[(K^2 - 1 - L\cdot I)/(K^2-1)]\,[-(L\,z^{+2}/2) - Jz^+] \tag{18}$$

where

$$L = \frac{4(Re_i - Re_o)}{(1-K^2)(H-G)}$$

$$I = \left(\frac{1}{6} - \frac{3G}{8} + \frac{G^2}{4} - \frac{K^2}{2} + \frac{K^4}{2} - \frac{K^6}{6} - GK^2\ln(K) + \frac{GK^2}{2} + \frac{GK^4\ln(K)}{2} - \frac{GK^4}{8} - \frac{G^2K^2[\ln(K)]^2}{2} + \frac{G^2K^2\ln(K)}{2} - \frac{G^2K^2}{4}\right)$$

$$J = 2/(H-G)$$

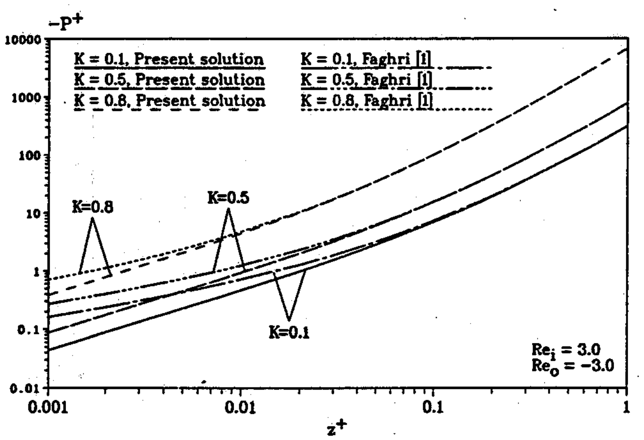

Fig. 1 Nondimensional pressure distribution along the annulus for different K values.

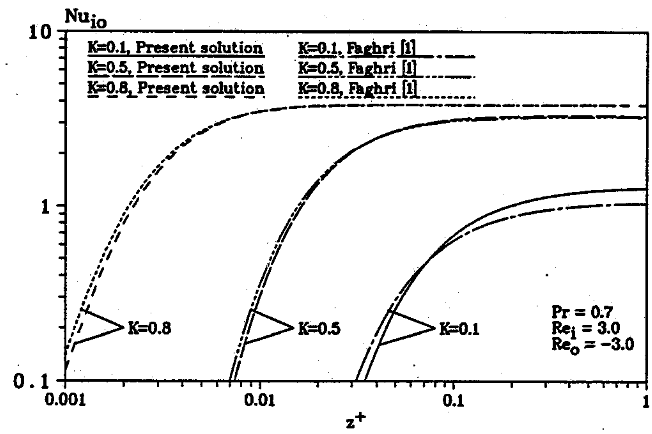

Fig. 3 Inner wall Nusselt number variation along the annulus for different K values.

Table 1 Dimensionless axial length z^+ at which dimensionless pressure P^+ from present solution is within 3% of the numerical solution by Faghri[1]

	$K = 0.1$	$K = 0.5$	$K = 0.8$
Re_i	z^+	z^+	z^+
0.0	0.775	0.200	0.0336
1.0	0.294	0.115	0.0263
3.0	0.196	0.060	0.0187
6.0	0.195	0.036	0.0139

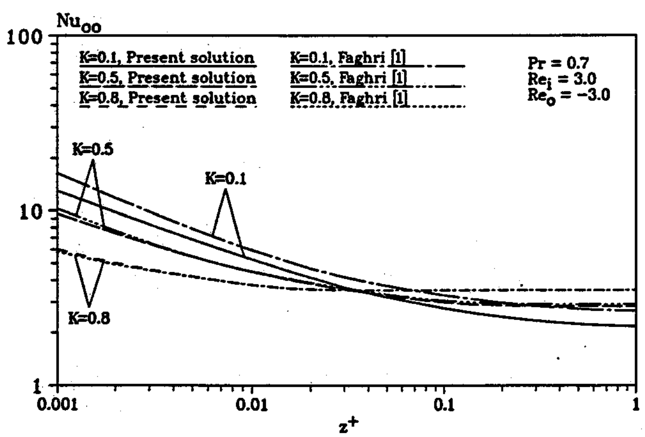

Fig. 2 Outer wall Nusselt number variation along the annulus for different K values.

The dimensionless pressure distribution given by Eq. (18) is shown in Fig. 1 for radial Reynolds numbers of $Re_i = 3.0$ and $Re_o = -3.0$ and K values of 0.1, 0.5, and 0.8. The numerical results given by Faghri[1] for the dimensionless pressure distribution in the hydrodynamically developing region are also shown for comparison. It can be seen that the agreement is excellent past certain values of z^+ that are dependent on the radial Reynolds numbers $|Re_i| = |Re_o|$ and the radius ratio K. Values of z^+ where the present solution of the dimensionless pressure is within 3% of the exact numerical results reported by Faghri[1] is given in Table 1 for different values of $|Re_i| = |Re_o|$ and K.

Equations (15–17) were also employed to solve the energy equation using the simplified Gaussian elimination procedure.[1] The heat transfer results in the hydrodynamically fully developed region for K = 0.1, 0.5, and 0.8 with $Re_i = 3.0$, $Re_o = -3.0$, and $Pr = 0.7$ are presented in Figs. 2 and 3 along with the results for the combined thermal and hydrodynamic entry lengths.[1] For K = 0.5 and 0.8, the results coincide in the fully developed region and also predict the heat transfer in the thermal entry region with surprizing accuracy (<2% error), compared to the results for the combined thermal and hydrodynamic entry length. This is due to the fact that as $K \to 1$ the flow becomes hydrodynamically fully developed very close to the entrance of the annulus. Therefore, the difference in the heat transfer due to the fully developed profile and the uniform profile is not significant in the thermal entry region. For K = 0.1, the results obtained by using the fully developed velocity profiles are within 22% of the results for the combined thermal and hydrodynamic entry length in the thermally fully developed region. Since the solution of the thermal entry length with fully developed flow is much simpler than that with the combined hydrodynamic and thermal entry, the fully developed axial and radial velocities presented here can be used to approximate the heat transfer in annuli with blowing at the walls in the thermal entry and thermally fully developed regions. Also, the pressure drop can be predicted in the hydrodynamically fully developed region by using Eq. (18) without numerically solving the nonlinear governing equations with different blowing specifications at each wall, as was done by Faghri.[1]

References

[1]Faghri, A., "Heat Transfer Characteristics in Annuli with Blowing and Suction at the Walls," *Journal of Thermophysics and Heat Transfer*, Vol. 4, 1990, pp. 59–66.

[2]Faghri, A., and Thomas, S., "Performance Characteristics of a Concentric Annular Heat Pipe, Part I, Experimental Prediction and Analysis of the Capillary Limits," *Journal of Heat Transfer*, Vol. 111, 1989, pp. 844–850.

[3]Faghri, A., "Performance Characteristics of a Concentric Annular Heat Pipe, Part II, Vapor Flow Analysis," *Journal of Heat Transfer*, Vol. 111, 1989, pp. 851–857.

[4]Kays, W. M., and Crawford, M. E., *Convective Heat and Mass Transfer*, 2nd ed., McGraw-Hill, New York, 1980.